# Thoughts on superglue 

Frank Mittelbach<br>2003/05/16 well 17 by now + thoughts from 19th + thoughts from 2003/06/02

the priorities are modelled by an 'infinite sequence of lengths' with all but a finite number being the same (ie the last length in the spec is repeated as needed) and for shrink this repeated length must be zero;

I think this statement is correct even though it may need reformulation in a real life implementation :-) but as far as the model goes I buy into it. The only correction I would make to the above is thet "length" here refers to something that $\mathrm{T}_{\mathrm{E}} \mathrm{X}$ allows to put after the plus (or minus) in a \skip, i.e., it may have a $\operatorname{fil}(1(1))$ component.

1. What does 'normal glue' look like when specified as superglue?
2. How does a glob of superglue (or the 'sum' of all the globs in the box) look like 'normal glue' to the box-badness calculator.

To answer the first question: A normal glue is modelled by specifying its stretch devided by a magic integer number CP (see below) as the first priority stretch and no other priorities (which means the first one is repeated ad infinitum). As far as i can tell at this time of the day after 10 hours work :-( this should result in exactly the same behaviour as now (if only normal glue is used). Only the process to calulate the ratio will be somewhat slower since different.

To answer the second question: it does not, at least not in my mind. Or say it differently: the statement in the draft paper from January is misleading (if not wrong):

Superglue is only use to distribute the white space within a box; is not used in calculating break point badnesses.

It is used alright, at least in the following the sense (also from January paper):
Starting at the first priority determine whether the superglue present can exactly fill the required white space.

If it can't go to the next level and repeat.

In other words, during the first step of the badness calulation superglue is certainly present and being used to determine the badness in so far as the first step is to distribute the white space and this is now more complicated than before.

And as a consequence I think you need to change the badness calulation otherwise the result is not really interesting. In particular the concept of a "total stretchability" needs some updates. Let me try:

With normal glue, a box has a 'total stretchability' that is used to calculate the glue-set-ratio and this is the sum of the 'stretch' of the globs of glue in the box; how is the 'total stretchability calculated for superglue (it is probably still the sum of something from the individual globs, see below)?

I think when you wrote this you had in mind the way how "glue is set" eventually, i.e., by multiplying with the a glue ratio and for that one needs two numbers. I don't think that you can keep that approach.

So let's see if i can write up what i have/had in mind.
TSP (total space) in a box is defined to be a superglue structure in which all superglues in the box have been added up on each priority level individually. TS is then defined to be TSP with the natural size being set to zero.

That can be easily built up while going along so...
According to our rule of not adding to the order of complexity of the calculations/structures needed, this ' TS of a box' needs to be a function of some field of the individual globs of superglue that can be easily calculated as the glob is processed whilst building the box (most likely by addition) before knowing what the final width of the set box will be.
... is no problem, the problem is that this is not a TS that you can stick into the badness function the way you know it.

And this is where in the simplest version something I call a "cutoff priority" (CP) comes into play. The CP is the priority level up to which priorities are all considered to be part of the "natural stretch" (or shrink but we may actually have both a CSP (cutoff stretch priority) and a CSP (oops, a cutoff shrink priority:-)).

This is why the "normal glue" is divided above by that number since we then have CP priorities with "normal glue" $/ \mathrm{CP}=$ "normal glue" being the natural stretch for that glue.

In that case the statement:
Thus the 'TS of a box' cannot depend on the 'priority levels' that actually get used when allocating the space.
would probably be true again.
There are however a number of problems with this approach if taken blindly, eg, suppose you want to stretch by 10 pt you have $\mathrm{CP}=2$ and a TS of $\mathrm{TS}=3 \mathrm{pt}$ / 5pt / 0pt (ie only superglues that only stretch to a certain point but never
beyond). In that case you simply can't reach those 10pts. Thus taking the sum of the of all prios not exceeding CP in TS and feeding that sum as TS' into the badness calulator is perhaps not really good.

Of course there are some ways to mend that, one could be to change the definition for superglue so that always

$$
\mathrm{TS}_{\mathrm{CP}+n}=\sum_{i=1}^{\mathrm{CP}} \mathrm{TS}_{i}
$$

for all $n>0$ and perhaps that is really good enough for what one needs, but on the other hand i think one can work with the more general TS but then I really think the calculation needs some changes-but i'm too tired to actually think it through how.

## 1 Chris' examples from 2003/05/19

### 1.1 Set up

Notation (no explicit units) for strtch of a superglue spec:
level1 max / level2 max / ....
Two SG specs:

$$
\begin{aligned}
G A & \equiv 4 / 2 / 2 \\
G B & \equiv 0 / 4 / 2
\end{aligned}
$$

Some boxes. For any reasonable $k$ :
The Box called $X k A$ needs $k$ units and contains just one $G A$. The Box called $X k B$ needs $k$ units and contains just one $G B$. The Box called $X k A B$ needs $k$ units and contains just one $G A$ and one $G B$.

The badness of box $X X X$ is denoted $b(X X X)$.

### 1.2 Exercises

Is any of the following statements clearly true or clearly false?

1. For $k \in[1,6]$ (or even for all $k$ ), boxes $X k A$ and $X k B$ have equal badness.
2. $b(X 6 A)=b(X 4 B)$
3. $b(X 5 A)=b(X 2 B)$
4. $b(X 4 A)=b(X 0 B)$
5. $b(X 2 A)=b(X 2 A B)$
6. $b(X 4 A)=b(X 4 A B)$
7. $b(X 6 A)=b(X 6 A B)$
8. $b(X 8 A)=b(X 8 A B)$
9. $b(X 6 A)=b(X 6 A B)$

Put the following in order:
$b(X 1 B), b(X 1 A), b(X 1 A B), b(X 2 B), b(X 2 A), b(X 2 A B), b(X 4 B), b(X 4 A)$, $b(X 4 A B), b(X 6 B), b(X 6 A), b(X 6 A B), b(X 8 B), b(X 8 A), b(X 8 A B)$

## 2 Chris on total glue in a box from 2003/05/24

Some possible partial solutions (not unrelated to each other).
As usual, simplify to stretch only for the discussion.

### 2.1 Each superglue has a normal TS

This is just like making each superglue also a normal glue; but its normal-TS is used only in calculating the TS, and hence the badness, of a box.

The normal-TS could be explicitly specified or deduced (somehow) from the superglue spec.

Problems What is the value of this TS?

### 2.2 There is a single total-superglue for a box

This total-superglue has the same structure as a glob of superglue and the badness of the box depends only on this single total-superglue structure.

Also, the total-superglue can be simply constructed incrementally from the individual superglues.

Problems Two related questions:
What is the badness of a total-superglue? (Equivalently, any superglue).
What is a 'good' incremental construction of the total-superglue?

## 3 Chris on badness of superglue

Note: I am writing about badness as I find it more useful; but I am still thinking of it as being 'the cube of a ratio of lengths' so this can all be rephrased in terms of two lengths: 'required stretch' and 'available stretch'.

So we have two possible questions:

What normal glue is equivalent to a superglue?
What is the badness of a superglue, given a construction method for the total-superglue?

The first is just a subquestion of the second in that its assumes some partial decision has already been made about the more general second question.

Since it is not clear that there is an obvious answer to the more restricted question we may as well tackle the more general one, thus.

### 3.1 Simple additive superglue

In this case the total-superglue is simply the component-wise sum of the individual globs: simply add up within each level. This means that there is clearly no distinction between globs of superglue and total-superglue. (If there is only one glob of superglue in the box then they are identical.)

Problems One problem with straight additivity is that for each superglue it confounds the following separate ideas:
maximum at a level;
contribution to badness calculation (ie how bad it is to use space from that level).

### 3.2 Weighted badness superglue

In this superglue structure each level contains both a maximum and a measure of how bad (relatively) it is to use this level of stretch.

Problems How to incrementally combine (or in any way combine) such relative badness weightings (RBW).

This is very closely related to the fundamental problem I keep coming back to but have only vague feelings about solving:

Find a good way to define the badness-function of a superglue spec (either the simple one or one with weighted-badness).

This RBW could be given by another length 'the base stretch at that level' at which the contribution to badness from this level would have particular global value (eg 100, or 10??).

Then incremental combination of RBWs could be by simple addition of these base stretches.

But I am not sure that this is a good model and it is getting rather complex.
Frank's CP idea may be all that is needed but I do not see how this will work right now or how it works for the examples below.

## 4 Specifications

A superglue is defined by its natural space $n$ its extra space $e=e_{1}+e_{2}+\cdots+e_{c}$ where $c$ is a constant per document design. and its stretch space $p$. Each $e_{i}$ can be zero.

The formula for calulating the badness $b$ for a box with one such superglue needing $s$ units of space can be calulated as follows:

$$
b= \begin{cases}0 & s=0 \\ 100\left(\frac{s}{e}\right)^{3} & e \geq s>0 \\ 100\left(1+\frac{s-e}{p}\right)^{3} & e<s>0 \text { and } p>0 \\ \infty & e<s>0 \text { and } p=0\end{cases}
$$

Right now assumption $e \geq 0$ and $p \geq 0$ but probably not a necessary condition.

Normal glue, e.g., plus part being $q$ would be specified as superglue as

$$
e_{1}=e_{2}=\ldots=e_{c}=\frac{q}{c} \quad p=q
$$

That gives

$$
b= \begin{cases}0 & s=0 \\ 100\left(\frac{s}{q}\right)^{3} & p>0 \\ \infty & p=0\end{cases}
$$

Now for your questions:

1. For $k \in[1,6]$ (or even for all $k$ ), boxes $X k A$ and $X k B$ have equal badness. False: since for $k \in[1-4]$ :

$$
100\left(\frac{k}{6}\right)^{3} \neq 100\left(\frac{k}{4}\right)^{3}
$$

i hope :-) and for $k \in[5-6]$ :

$$
100\left(\frac{k}{6}\right)^{3} \neq 100\left(1+\frac{k-4}{2}\right)^{3}=100\left(+\frac{k-2}{2}\right)^{3}
$$

and for $k \in[7-\infty]$ :

$$
100\left(1+\frac{k-6}{2}\right)^{3} \neq 100\left(1+\frac{k-4}{2}\right)^{3}
$$

2. $b(X 6 A)=b(X 4 B)$

True because

$$
100\left(\frac{6}{6}\right)^{3}=100\left(\frac{4}{4}\right)^{3}
$$

3. $b(X 5 A)=b(X 2 B)$

False because

$$
100\left(\frac{5}{6}\right)^{3} \neq 100\left(\frac{2}{4}\right)^{3}
$$

4. $b(X 4 A)=b(X 0 B)$

False
5. $b(X 2 A)=b(X 2 A B)$

False because

$$
100\left(\frac{2}{6}\right)^{3} \neq 100\left(\frac{2}{10}\right)^{3}
$$

6. $b(X 4 A)=b(X 4 A B)$

False
7. $b(X 6 A)=b(X 6 A B)$

False because

$$
100\left(\frac{6}{6}\right)^{3} \neq 100\left(\frac{6}{10}\right)^{3}
$$

8. $b(X 8 A)=b(X 8 A B)$

False
9. $b(X 6 A)=b(X 6 A B)$

False
Put the following in order:

$$
\begin{aligned}
& b(X 1 A B)=100\left(\frac{1}{10}\right)^{3}=0.1 \\
& b(X 1 A)=100\left(\frac{1}{6}\right)^{3}=0.4629 \\
& b(X 2 A B)=100\left(\frac{2}{10}\right)^{3}=0.8 \\
& b(X 1 B)=100\left(\frac{1}{4}\right)^{3}=1.5625 \\
& b(X 2 A)=100\left(\frac{2}{6}\right)^{3}=3.7037 \\
& b(X 4 A B)=100\left(\frac{4}{10}\right)^{3}=6.4 \\
& b(X 2 B)=100\left(\frac{2}{4}\right)^{3}=12.5 \\
& b(X 6 A B)=100\left(\frac{6}{10}\right)^{3}=21.6 \\
& b(X 4 A)=100\left(\frac{4}{6}\right)^{3}=29.629 \\
& b(X 8 A B)=100\left(\frac{8}{10}\right)^{3}=51.2 \\
& b(X 4 B)=100\left(\frac{4}{4}\right)^{3}=100 \\
& b(X 6 A)=100\left(\frac{6}{6}\right)^{3}=100 \\
& b(X 6 B)=100\left(1+\frac{6-4}{2}\right)^{3}=800 \\
& b(X 8 A)=100\left(1+\frac{8-6}{2}\right)^{3}=800 \\
& b(X 8 B)=100\left(1+\frac{8-4}{2}\right)^{3}=2700
\end{aligned}
$$

